GENESIS OF NUMBER-FORMS.

By D. E. Phillips, Clark University.

Before entering into the results of this inquiry it may be well to make clear the nature of these forms and to give a short account of former investigations. The first is a very difficult task, for psychological phenomena that appear foreign to us are hard to comprehend, especially when so strange. The reader will be greatly assisted in what follows by first examining carefully the drawings given on pages 512 and 513.

The general character of a number-form is such that whenever a number is thought of, it appears in the same place on a visual diagram which is invariably called up, viewed by the mental eye, often definitely located, and which usually consists of an irregular composition of lines on which the figures appear either written or printed. These diagrams are often enormously large in comparison with the drawings here presented. "Sometimes," says Galton, "a form has twists as well as bends, sometimes it is turned upside down, sometimes it plunges into an abyss of immeasurable depth, or rises and disappears in the sky."[1] In some instances the line does not appear; nevertheless, the numbers occur in a fixed order, but are usually less complicated. Galton says the most common way is to see only two or three figures of the diagram at once, but in my investigation, that depends upon whether the mind is performing mental calculations, or the form is viewed as a whole. The entire form can usually be seen, and, by many, as distinctly as if viewed by the natural eye.

"Number-forms," says Galton, "are in each case absolutely unchangeable except through a gradual development in complexity. Their diversity is endless, and the number-forms of different persons are mutually unintelligible. These strange 'visions,' for such they must be called, are extremely vivid in most cases, but almost incredible to the vast majority of mankind, who would set them down as fantastic nonsense;

[1] "Inquiries into Human Faculty," p. 123.

nevertheless, they are familiar parts of the mental furniture of the rest, in whose imaginations they have been unconsciously formed, and where they remain unmodified and unmodifiable by teaching."[1]

The number of individuals possessing such visual schemes depends upon what limitation is put upon the term "Number-Form." For example, some persons have a diagram for the days of the week, days of the month, or months of the year, etc., who have no number diagram. These are evidently of the same nature, and if they be included in our estimate of forms the ratio is much changed. Again we find all degrees of clearness, and some number-forms appear as all other mental imagery, or fade away until classification becomes difficult. So here, as in most fields of investigation, differences on many points result largely from the differences in the extent given to the subject by the various investigators. Surely this will to a great degree account for the variations found in the reports of those who have investigated this subject.

Galton, who published the first article on these forms in *Nature* and afterwards in "Inquiries into Human Faculty," states that "the peculiarity in question is found, roughly speaking, in about one out of every thirty adult males, or every fifteen females."[2] But he considers only clear number-forms and the estimate is on adults. The next work was done by G. T. W. Patrick of the University of Iowa, and appeared in The *Popular Science Monthly*, Feb., 1893. He is inclined to think that one out of every six adults would be a more accurate proportion, that the proportion among children is greater, and that it is perhaps a little more common among women than among men.[3] It is to be observed, however, that among the diagrams which he gives are diagrams for the months, days of the week, seasons, and alphabet; no mention is made of whether they are counted in the estimate of one in six.

The same year Flournoy published his *Des Phénomènes de Synopsie*, in which he includes number-forms. He informs us that he received returns from 370 persons between 18 and 40 years, and found that in childhood colored-hearing or *photisme* is much more frequent and gradually diminishes, while on the other hand schemes are much more stable and endure through life from the time they first exist. At 20, 1 in 6 have colored-hearing; 1 in 9 have visual schemes, and

[1] "Inquiries into Human Faculty," p. 156.
[2] "Inquiries into Human Faculty," p. 119.
[3] *Pop. Science Monthly*, Feb., 1893, p. 506.

1 in 15 have both.[1] About the same time Miss Calkins published in THE AMERICAN JOURNAL OF PSYCHOLOGY her examination of 543 students, all girls, of whom 78 have forms, 32 colored-hearing, and 14 have both.[2] In the fall of the same year she examined 203 new students, finding 32 with colored-hearing, 61, or 30%, with forms, and 17 with both.[3] In the first case about 1 in 7 have forms, and in the last nearly 1 in 3. Here all of the various diagrams mentioned are counted. Out of 300 children, from 10 to 12 years of age, Binet found only 3% with number-forms, but attributed the result to his inability to make them understand what he desired and to their indifference.[4] These variations are largely due to the different limits given to these psychological phenomena, the difference in age and sex of subjects investigated, and the manner of investigation.

The present writer's interest in this subject was first aroused by some strange and complicated answers to the following questions in a syllabus on Number and Mathematics: (*f.*) Cases of number-forms, *e. g.*, the first 12 numbers being habitually associated with a dial or clock-face, the first ten on a line, straight or curved, systems of dots, colors, etc. Do odd seem to you different from even numbers? Draw any number-forms. How do you arrange days of the week or month, the musical scale?" As this section did not cover the points of greatest interest, and as each seemed to have great difficulty in explaining these mental forms in writing, I began a personal examination of 332 Normal School students. Still some questions of importance were overlooked in the early part of the work. The following questions were finally formulated and the investigation extended to 974 school children of Worcester, Mass., and 343 miscellaneous adults personally interrogated: 1. At what age did it appear? 2. How did it originate? 3. Is it useful, or troublesome? 4. Do you see the figures on a line? 5. How large does it appear to be? 6. Where is it located? 7. Are you left-handed? 8. Do you know of any forms in your family? 9. State any peculiarities about your form or its use. 10. Do you like mathematics? 11. Give name, age, and sex.

The following table shows the sources of the material for this article, number of forms collected, the per cent. having some form, and the per cent. having number-forms.

[1] "*Synopsie*," p. 15.
[2] AMERICAN JOURNAL OF PSYCHOLOGY, Vol. V, p. 265.
[3] *Ibid.*, p. 439.
[4] "*Synopsie*," p. 16.

TABLE I.

SOURCES OF MATERIAL.	No. Examined.	Number-Forms.	Month-Forms.	Week-Forms.	Alphabet-Forms.	Lord's Prayer, Songs, etc.	Total Forms.	Total Having Forms.	Per cent. Having some Form.	Per cent. Having Number-Forms.
Normal School,	332	20	45	13	3	—	81	59	18—	6
Miscellaneous,	343	23	20	8	7	4	62	45	13+	7—
Returns on General Syllabi,	360	25	31	2	4	—	62	50	14—	7
School Children of Worcester, 10-16 y.	974	79	118	11	9	—	217	167	18—	8+
Total,	2009	147	214	34	23	4	422	321	16—	7+
Males,	969	67	74	19	13	4	177	145	15—	7—
Females,	1040	80	140	15	10	—	245	176	17+	8—

The 332 Normal students I examined personally, and had opportunity to see some of them a year later, but found no change of any note. Fully 80% of this class are teachers. The 343 miscellaneous are also cases personally investigated. Nearly all of them are adults over 25 years old; 92 are teachers; 41 Clark University men; the remaining persons are, generally speaking, uneducated. A few cases were obtained by correspondence. Many answering the syllabi omitted points of importance, and some not included here simply spoke of the direction numbers take in their minds. The returns from the Worcester schools were collected from the 7th, 8th and 9th grades, from children of 10 to 16 years old. After a short explanation the children were asked to draw whatever form, or forms, they had. In all the rooms, except five, an effort was made to keep them from obtaining an imaginary form for the occasion, and in only a few cases was there reason to think the forms were not genuine; each pupil giving a form was, as a rule, questioned privately. In

the five rooms fair opportunity to copy or devise a form was offered pupils by presenting drawings and entering into detailed explanation; but the forms collected there show no signs of fraud. The percentage of forms was not quite as large in three rooms as is usually found. After giving the slightest explanation, a close observer will hardly fail to distinguish every one having distinct number-forms. Those who have no form have no idea of what you are speaking of, and are often slow to comprehend any explanation, appear surprised or treat indifferently what you say. Those having a form show an entirely different attitude. In a room of 44 children the mere mention of forms showed four who comprehended my meaning, and they were afterwards found to be the only ones having forms. Six men in a shop were asked if any of them had a number-form. One asked, "What do you mean?" On a word of explanation he exclaimed, "I have the craziest thing you ever saw," and proceeded to outline No. 12, Plate II. While at their play I asked two girls, about five or six years old, if they could count one hundred. Each replied that she could. When asked which way the numbers go, one replied, "They go round and round, then up to the clouds," making a circling motion with her hand. In the first grade I examined 23 children privately, finding two clear number-forms. The first boy was a little over six and had just learned to read. While he was counting 100 I observed that at certain points there was considerable struggle, as if he had lost sight of something. When asked if the numbers went down, he shyly said, "They go up and turn," and then drew on the board Form 17, Plate I. I tried to deceive him by placing numbers on the turns to suit myself; five different numbers were placed where 17 is, but each time he shook his head; finally he told me to put 17. He located all the others with the same certainty. Later the form was slightly changed; the boy was called and asked if it were correct. He is very shy and slow, but soon proceeded to correct it even to the length of the lines, which he was not tall enough to draw himself. The clearness and persistency of such a form are not to be doubted.

Some will be inclined to mistrust results obtained from children from 10 to 16 years, and while every precaution was taken to avoid error, I do not consider that this part of the study is free from such. But it cannot be far wrong, for almost the same ratio exists among adults, none of whom remembers a time when his form did not exist. The ratio, however, is, as we shall see, of little importance, and the culminating point of this work lacks no reliable evidence.

A few things in Table I are worthy of attention. The ratio

for number-forms remains much more constant than for other forms, while children have more week and month forms than adults. The total number of forms is nearly one-third larger than the number of persons having forms. This shows that several have more than one form of some kind. Where a distinct number-form is found, the individual often has other forms, but many that have a month-form, week-form, etc., have no number-form. The last two columns show that the per cent. having some form is more than twice as large as for those having only number-forms, that the per cent. decreases a little with age, being about 1 in 12 for children and about 1 in 15 for adults. The proportion seems to be a little greater among females than among males; especially is this true for month-forms. Omitting these there is by no means so great a difference as Galton found. As a rule men think such things very trivial and are not as willing to respond as women. As we hope to show later, unless an arbitrary definition and limit be given to forms, a table of this kind can be no accurate guide. The records here are based upon comparatively distinct forms. Since completing this table several returns have been received that might change the ratios somewhat. Prof. Barnes, Fairmount Normal, W. Va., states that on examining 118 boys and 136 girls, he found that 69 boys and 53 girls had some form. But there was no personal examination and no forms drawn.

TABLE II.

General Direction, First Turns, and Endings.

	No. of Forms.	Turn.					End.		
		To Left.	To Right.	At 10	At 12	At 20	Below 100	At 100	Above 100
Galton,	65	23	38	15	27	5	16	26	22
Flournoy,	37	4	29	14	5	5	10	14	12
Patrick,	14	2	9	2	0	0	5	1	5
Present Collection,	147	18	91	60	32	11	78	53	16
Total,	263	47	167	91	64	21	109	94	55

Table II gives a classification of the principal direction,

Plate 1.

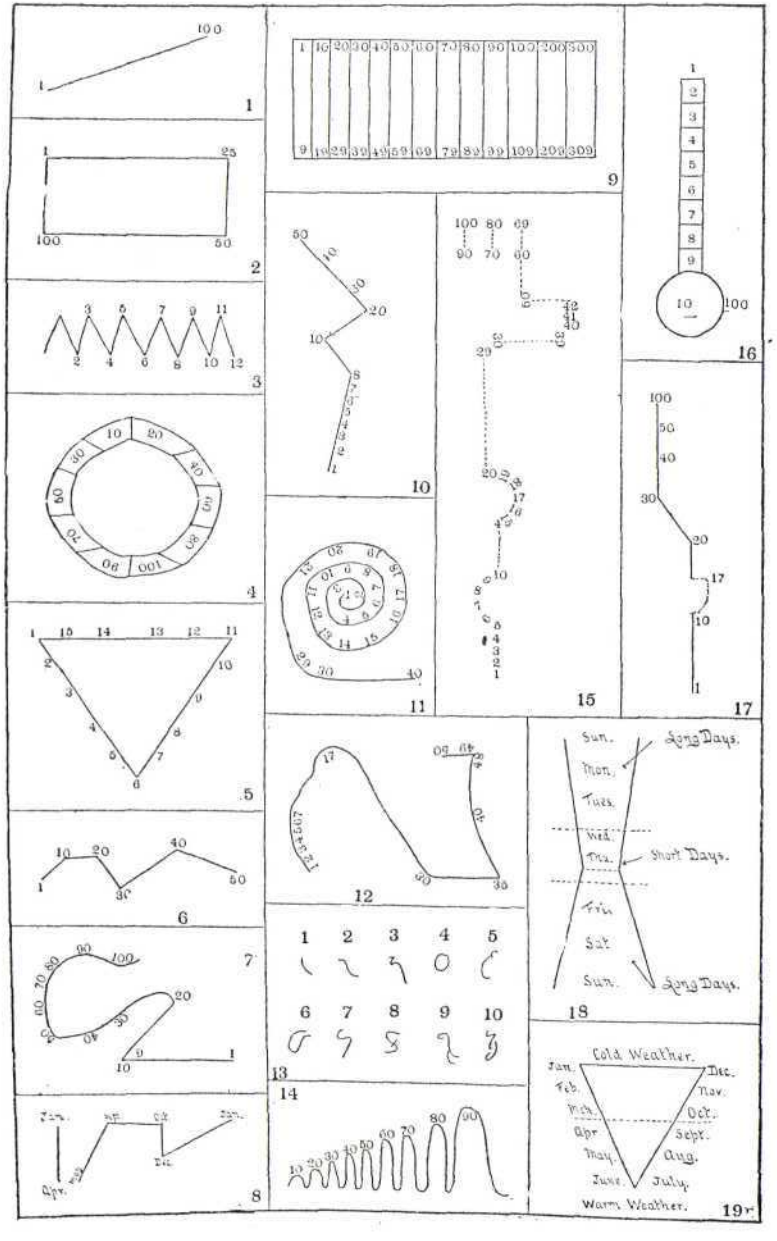
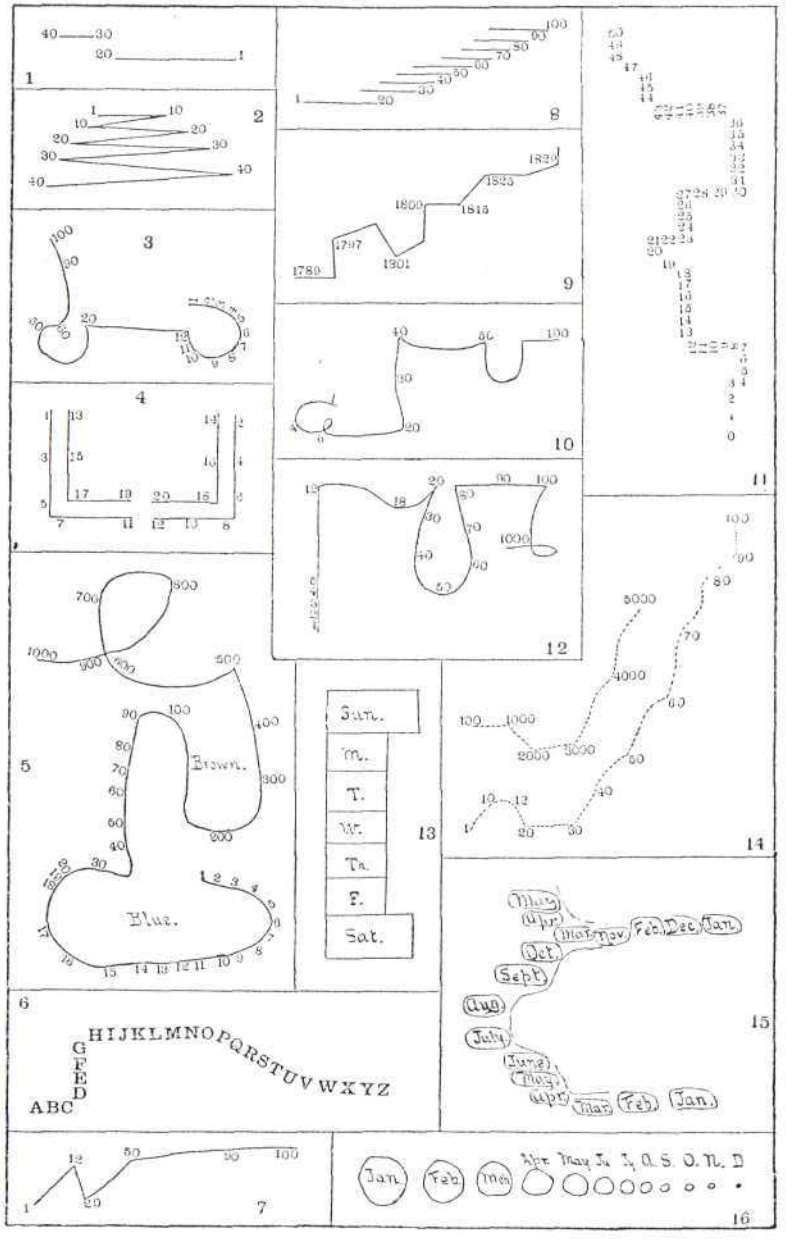


Plate 2.



first turn, and ending, of all the number-forms I have been able to find published, including my 147. The direction and first turn have been referred to by others, but no definite summary made except that Miss Calkins directs attention to an outline of turns. Her work is not included here because information on other points was wanting and few forms were published. Out of 68 forms she gives 17 turning at 10, 8 at 12, and 5 at 20.[1] The above table is but a rough classification, but it shows some tendencies in number-forms for which the psychologist may be puzzled to find an explanation. A glance at the total directions and turns shows that these do not include all the forms, for a numeral scheme may take any direction and make its first turn at *any* number, but these show such majority as to point to some general laws. Very few forms go down, but some are so complicated in direction that classification is impossible. The endings are only approximately correct, for many do not draw the entire form, and some forms are capable of almost indefinite extension according to the series in mind. We shall have occasion to refer to the lines here indicated later.

Plate I shows the leading types and some of the most highly developed forms of persons under eighteen; Plate II contains only adult forms. These two plates indicate that number-forms develop in complexity, and perhaps change somewhat in shape, notwithstanding that most individuals think that their form has always been substantially the same. This contrast and the three leading types, to be mentioned presently, would appear more striking if the entire collection could be published.

It seems almost impossible to classify these number-diagrams. Such a classification is perhaps of not very much value, and must be based upon other points as well as on the drawings. The variety and complexity of forms are wonderful. Month-forms are frequently alike, but I have found no two number-forms exactly the same. Still, taking several points into consideration, we may note three rough types which include nearly all number-forms. Nos. 1, 2, 5, 9 and 16, Plate I, and Nos. 1, 4, 8, Plate II, represent the simplest that it seemed advisable to include, although there is really no line of demarcation. The chief characteristic of this class is the evidence of conscious construction, or of deduction from, or alteration of, some form or object more or less frequently before the eye. Of 19 who have suggested an orgin for their number-form, 15 belong to this class. As regards localization, vividness, etc., these individuals are, as

[1] AMERICAN JOURNAL OF PSYCHOLOGY, Vol. V, p. 449.

a rule, most uncertain. The fact that they frequently locate the forms on blackboards, charts, books, blocks, etc., is evidence of such an origin, and of the fact that they are slightly modified mental images. One lady says she is conscious of taking hers from blocks on which the numbers were stamped; and she is the only one who has complained of a form being troublesome. Another is conscious of arranging the numbers on the edge of a chart and ever afterward having the form. This will explain many that take a real geometrical form. As a rule this class does not exist to any great extent among adults, and since adult forms can hardly be modifications of these, it is probable that they drop out early in life and that those formed most unconsciously are most persistent. They are never so striking to the individual nor used with half as much freedom and elasticity as those that seem more spontaneous and to others more unmanageable.

Nos. 3, 6, 8, 10, 15 and 17, Plate I, and Nos. 2, 7, 9, 11 and 14, Plate II, furnish examples of the largest class of forms. They are usually pronounced useful, but not so vivid as the more complex ones. To this belongs Diamandi's mentioned later. More than half the forms collected belong to this class, and the similarity between them is sometimes considerable.

Nos. 7, 11 and 12, Plate I, and Nos. 3, 5, 10 and 12, Plate II, show a decided tendency to take a circular or curved direction. They are usually exceedingly vivid, strangely located, and generally said to be useful in a manner and to a degree wholly unintelligible to any one except the possessor. There are but few of these and they usually extend beyond 100.

Besides these three classes there are some very exceptional forms. C. C. S. and L. B. have forms of three dimensions. Mr. L. of Clark University tried several times to draw his, but could not. He finally described it as resembling the threads of a screw about four feet in diameter, the threads enlarging on the far side, and after the first round running almost parallel. The first one hundred completes a round, and each succeeding hundred a little more than a round, ending directly over 2, 3, 4, 5, etc., respectively. If two thousand is thought of as twenty hundred, it appears on its proper round over twenty, but if as two thousand, it is located on the same round, but over two. The form as a whole is located on a plain in front, goes to the left, and sometimes seems to extend behind him. It is very definite and clear, and highly useful in all numerical calculations. No. 13, Plate I, was obtained from a Swede boy of 17. I

could not understand the peculiar characters under the numerals; two personal interviews gave no information except that these are directions which the numbers take when he hears them, and that the same feeling is received when he sees them, unless they are passed over rapidly. The boy is timid and slow to learn. The characters are reduced to about one-eighth of their original size. Later a girl of fifteen presented a similar case. Her characters do not resemble those of the boy, and are somewhat larger. She gives the same explanation, except the power to receive such a sensation from sight is gradually diminishing. She is an American girl of average intelligence.[1] These two cases are peculiar, and call to mind an observation made some ten years ago, on a boy about 12, in my school, who said that each note had a peculiar swing to him when spoken, and when singing he had an impulse to follow the same.

Let us now examine the answers to the questions asked. Of 321 giving forms, 280 answered the question: "At what age did it appear?" 241 do not remember a time when it did not exist; 17 feel sure that their number-form originated about the age of 6; 22 place their month-form after 12 years; 11 say they learned to add and multiply on their scheme. Prof. Patrick gives quite an extensive form of a girl 9 years old; No. 7, Plate I, is from a girl only 6, and No. 17 has already been mentioned. Nothing is more marked than the very early period at which these schemes are developed. Probably number-forms begin with the naming of numbers, and go on as the child learns to count one hundred. There are several things in the forms that point to this fact. Most children can count one hundred before they learn to recognize anything written or printed. This begins at a period when, of all others, the largest part of our knowledge comes through sight; it is only natural that they should try to visualize the abstract and to cast it into some concrete form or space relation. Table II shows that of 263 forms 91 make their first turn at 10; 64 at 12, and only 55 extend beyond 100. There is at least a suggestion that both the first turning points and the limits of these forms may be related to the manner of counting, and indicate that their formation begins with the earliest counting. A child is generally taught to count 10 on its fingers, yet this is quite different from its previous knowledge, which chiefly consists in learning the names of things. It by no means recognizes the numbers as the names of its fingers. Generally there is

[1] Cf. the somewhat similar diagrams given by Miss Calkins,—AMERICAN JOURNAL OF PSYCHOLOGY, Vol. V, Fig. 7, Plate I, facing p. 464.

a pause in the counting process when ten or a dozen is reached, then it is again taken up only to plunge the child into pure abstraction, and continued until 100 is reached. A friend writes, "F., 4 years old, does not know his letters, but he can count 100." Learning to count before learning characters of any kind is the rule, almost without exception, among country children. Of 1160 individuals, including 660 children, 280 high school students, and 220 adults, 92% learned to count first; over 80% assert that they could count 100 before they could read.

In this connection I wish to add some testimony on a point that will doubtless seem mystical and incomprehensible, but I cannot forbear to state the facts. When my work was presented at the seminary, the question was raised whether these forms could precede the power to recognize the written or printed figures. To my astonishment two possessing forms immediately declared that they were confident that their forms existed before they knew written or printed characters of any kind. After this I talked the subject over with Dr. Story, Professor of Mathematics at Clark University, who thought this impossible, and attributed it to their inability to remember accurately the period of absence of a thing so long and continuously in the stream of consciousness. However, since that time I have reached as many persons having forms as I could conveniently, and many who profess an early recollection of their forms give the same testimony. Besides this there are cases of individuals having forms for certain anthems, the Lord's Prayer, and Doxology. These are not mental images of a printed page, but a projection of an indiscernible something into space. Dr. S. sees the first phrases of the Lord's Prayer nearest; the remainder gradually recedes. This was learned early, while the creed, learned from a book, is reversed. In a like manner early number-forms do not give a visual image of any characters at first, but, so far as I can understand, simply a division of space in these pecular directions. Mr. T. states that his form existed as early as the fifth or sixth year, yet he did not know figures and was unable to read until he was ten. Mathematical prodigies have exhibited a great power to visualize numbers, yet in most cases the power was well developed before learning to recognize printed characters. It is here, as we hope to show later, that the best approach to the genesis of number-forms is to be found.

As already stated only 19 offered any explanation of the origin of their numeral scheme, and in most cases the explanation is not satisfactory even to the giver. Twenty-four give an explanation of their month-form. "Brother, 5 years

old, cannot add when not in a room with a clock. He counted by the hour spaces; he is now 7, and uses the clock face mentally." Lady of 26 years says, "I learned to count when 5 years old, but could not remember 11 and 12. My grandfather told me to count two 6's, and that made a dozen. For some time I calculated time and objects by 6's; never how many 12's, but so many clock-faces. I used to arrange pebbles in the sand as on a dial. I can't explain to anyone how I can count faster that way, but I can." Such a description may safely be trusted.

Is it useful, or troublesome? To this point particular attention was given because some investigators have considered such forms a hindrance to abstract thought. But only those who have no diagram make such an objection.

Mrs. Hornbrook has given in the *Educational Review* a review of some work in which she attempted to show the utility of number-forms. She gives there her own form, which has been of so much service to her. The numbers up to 100 were arranged in the form of a square. In some tables the 10's were put in very heavy type; in others the multiples of 5, 6, 8, 9, etc. These tables were always kept where they could be seen, with the hope of producing a form in the child's mind. She states that they were not able to discover any permanent forms, but that forms were developed and used while learning the multiplication table.[1] I am inclined to think that such forms were only localized memory of the tables, such as anyone can form, by a voluntary effort, of objects often seen, and doubtless closely related to the first division of forms already given. As to the utility of number-forms in many cases, I have no doubt. Of those having some form 211 answered this; 97 are sure of its utility; only one finds it troublesome; 113 say it is neither useful nor troublesome. From many I receive such replies as, "very useful," "could not add without it," "use it every day," "yes, I shut my eyes and count." Mr. B. says he cannot see how mathematical calculations can be performed without such. A girl of 18 states, "I could not add 7 and 9 without mine." Another, whose form appears on Plate I, No. 6, informed me that she put herself to sleep counting on it. Girl, 20: "In rapid counting I know at every notch on the line just what figure belongs there." "My form," says Mr. L., "is as necessary as bread and meat." In Miss Calkins' analysis of this point, about one-half do not think them useful, but she mentions no cases where they are considered detrimental. Now, those who are neutral are doubtless more nearly correct. They perhaps use their form just as we use language, without ever thinking that it is useful as a medium of thought. In fact, several who have said they did not use their form, have later corrected this statement by saying that they found themselves continually using it. In using this complex machine they always speak of "jumping" from place to place. The best way to understand this is to look at a surface, and then fixate the eye first on one point, then on another. This doubtless explains why in calculating many see only parts of the diagram. Fig. 3, Plate II, is a carpenter's. In telling how he used it he said: "When I reach 20 the horizontal parallel lines turn upright; the space between 10's is divided into equal parts, except the middle is larger for 5. If 9 were to be added to 37, which is now written in its place, I jump to 46."

[1] Vol. V, p. 479.

With the exception of Diamandi, who sees his on the convolutions of his brain, the rule is to locate in space near the head, when located at all. Several did not answer this question, and many were not conscious of any localization; but the most common replies were, "right in front of my eyes," "it is not located, but just stands out by itself," "I can see it hanging in the air right in front of me," "it always goes to the ceiling," "I can see it on the wall." Four localized their forms to the left about three feet, three overhead; some said it went up in front until they could hardly see it. When drawing his form one man found an ordinary sheet of paper too small, saying, "It ought to lengthen out," giving his hand a swing to the right. Flournoy gives the following cases: "M. Y.'s number-form, composed of parallel lines representing the hundreds, occupies the right half of the space in front of him. In the left half floats his diagram of the week in the form of a horizontal rectangular figure divided into seven bands, something like a leaf of ruled paper, floating in the air about a metre from him, opposite his left thigh. Still more to the left, and at the height of his head, is situated his year-form, an ellipse of small eccentricity presented in nearly a vertical plane. Whenever M. Y. thinks of a date of the year, of a day past or future of the current week, or of a number, he perceives it in its proper place on the corresponding scheme. I have often had occasion to make him write down rapidly a series of figures at random. He follows what he is writing only with an indirect vision, like a hurried copyist who lets his hand work of itself, and will not lose sight of the page he is copying."[1] He gives also an account of a girl, 11 years old, who had a diagram for

[1] "Society for Psychical Research," Vol. VIII, p. 146.

the months situated directly over her head, entirely out of the field of vision, so high that she could not reach it by 6 to 8 centimeters. It always kept the same distance from her head. Another sees the future in front and the past behind him. In order to see into the past he must place himself back in his imagination beyond the event, saying it is impossible to visualize time which is behind him.[1]

The space in the forms between the turns after 20 are not usually filled out, but the numbers when thought of take their places in order. As a rule near the end the numbers seem to be very much crowded, but those who describe their form as extending up into the sky, or far away in the dim distance, have no such trouble. In some no line appears; in others the line followed is a dotted line which the individual is conscious of tracing; but in the majority the line is somewhat distinct.

From Table II it is evident that about four times as many go to the right as to the left. Having found two left-handed persons with forms running to the left, it was suggested that this might have something to do with the direction. Miss Calkins kindly re-examined her subjects for me on this point, but found nothing to substantiate it. In my further investigations I have met with the same result. So the direction must have some other explanation.

Galton thinks these forms hereditary, with even sometimes a strong tendency to reproduce the form.[2] But as strong a similarity of forms is seen in many not in any wise related. An examination shows two or three general types, no matter where they are collected. Miss Calkins' result is much in favor of heredity. Flournoy says all that can possibly be attributed to heredity is a *predisposition*.[3] My answers on this question were, children not included, 128. Six were sure of relatives having a number-form; 17, of relatives having other diagrams. I find no more signs of, nor reasons to look for, heredity here than in any highly developed power of imagination, memory, art, music, etc., all of which are much questioned since Weismann's theory of heredity has become prominent. Two in the same family may have like forms by mere coincidence.

Ninety-four of those having number-forms assert that they are fond of mathematics. "I worked at odd times," says one, "for two years on a geometrical puzzle, and finally got it." "I like mathematics," says another, "but think my

[1] "*Synopsie*," p. 183.
[2] "Inquiries into Human Faculty," p. 140.
[3] "*Synopsie*," p. 203.

form has nothing to do with it." "I love to solve problems mentally," is a common reply. Twenty-eight "hate" mathematics. There are so many things to modify one's likes or dislikes for mathematics that it seems to me this proves nothing one way or the other. In my work I have tried to ascertain whether they are more general among dull persons, or the more intellectually active, but so far I have been unable to detect a stronger tendency in the one class than in the other. Neither does it appear that they are more general with the imaginative. This differs from Galton's statement that "it is found among most imaginative persons."[1] More number-forms have been found among forty-one persons in Clark University than in any equal number elsewhere, eight well defined forms having been found. There are three of three dimensions. My observation among higher mathematicians has been too limited to draw any conclusions. No one can say that one would be better without a form, for, as it appears only when numbers are thought of, it does not influence the ordinary thought processes.

In this connection it will be interesting to *résumé* the comparison which Binet makes between Inaudi and Diamandi, two mathematical prodigies.[2] Inaudi was born in 1867 of a poor family. During pregnancy his mother frequently saw the recklessness of her husband, and, pressed by poverty, she calculated in her head means of economy; her days were spent with figures until she acquired a mania for counting. This is the statement of his brother. At the age of six Inaudi acquired a passion for counting; never used fingers, pebbles, etc., but did all with words, learned the names to 100 from his brother and then demanded more. He entered Paris at thirteen; could neither read nor write. He was presented at the Academy of Sciences in 1892, and in 1892-3 was examined more than fifty times. When an infant his head was so large that it was thought that he could not live. He learned to read seven years ago, converses but little, yet shows good natural intelligence. In calculation or reproduction of figures he repeats the numbers three times, seems perfectly quiet, and can carry on a conversation on other subjects, the only effect being to prolong the time. Binet thinks that he must have over 300 figures in his mind at the same time.

Diamandi was born in Greece in 1868; entered school at seven; was at all times first in mathematics; left school at sixteen; became a grain merchant, and here his mathematical talent greatly developed. He had fourteen brothers and

[1] "Inquiries into Human Faculty," p. 114.
[2] "*Psychologie des grands calculateurs et joueurs d'échecs*."

sisters; only one brother and one sister possessed a similar aptitude. His mother had a wonderful memory for everything. He has abandoned commerce; reads much; everything written on mental calculations; composes verse and romances, and knows five languages. In 1893 he presented himself at the Academy of Sciences, and was examined by the same commission that examined Inaudi. Binet has experimented with him fifty times, from three to five hours each time. He came to contest with Inaudi, and it is this that makes the account of Inaudi of value here. Inaudi is an exception among mathematical prodigies. He is, so far as understood, of the purely auditory type, while Diamandi is of the visual type. For some time Diamandi denied having a number-form and kept it concealed for more than two months. Once he said that the figures appeared to him on one of his cerebral convolutions placed to the front and left.

"The form and location of this image in relation to the individual, are the elements which volition can scarcely modify." His number-form is of the usual type, extending from left to right, of broken lines, and space more occupied at the beginning. Besides this, Diamandi sees all objects in the centre of a complex figure formed by a grayish-colored mass, enclosing a lighter spot. The thought of a house, a dog, etc., brings the image of such into the lighter space. He first presented to Binet a roll of paper with 2,000 figures on it, any one of which he could reproduce and locate without difficulty, or read them diagonally or otherwise. When hearing figures in French he encounters great difficulty, being obliged to translate them into his native Greek, and often makes many errors, but never fails when they are presented on paper. After a first look at the figures on paper, he closes his fists; puts them against his temple; bows his head; soon takes another look at the paper, then closes his eyes and begins the operation. With him the making of a visual image is the important thing, while Inaudi appears to make no mental representation. Diamandi desires the figures written in a square, and he always begins at the left, no matter how they are written. The multiplication of 65,879x2537 was accomplished in 3 min. 10 sec.

If interrupted by noise or questions, Diamandi loses the image then in mind and is obliged to reproduce it. No matter how many of the figures are variously colored, he reproduces them with their proper color. With a small number of figures Inaudi is more rapid than Diamandi, otherwise Diamandi surpasses him in rapidity and in extent of reproduction and calculation. Whether this difference is due to other causes than the existence of Diamandi's number-form, Binet does not consider; although it is certain that he in some way makes use of it, always locating the figures in their proper places in his form in a manner which only those who have such diagrams can comprehend. Then they are as plain as if written before the natural eye. I give these two cases here because of their bearing on forms and on mathematical prodigies in general. Unfortunately number-forms did not gain attention early enough to be studied in former mathematical prodigies. A few are known to have possessed true number-forms. Dr. Scripture says the great peculiarity of mathematical prodigies is the visual images of numbers which they always carry about in their minds. And among general returns there is mention of three children extraordinarily rapid in use of numbers and each has a well defined form.

Flournoy *résumés* several thories which have been held as to the explanation of the whole matter of *Synopsie* as follows: All are analyzable into two great classes. The first seeks the explanation in psychological association. The other declares it outside of ordinary association and seeks its explanation in physiological conditions, holding that the continuity of the central cortex permits excitations to radiate to different centres of localization. It is an exceptional anastomosis uniting nerve fibres or cells ordinarily separated. Flournoy holds that the principle of psychophysic parallelism will harmonize both theories. He classes all the phenomena of *Synopsie* under affective, habitual, and privileged psychical associations. Affective association is the general coloring which each sensation gives to every other sensation received at the same time, no matter how heterogeneous. This is only natural from the unity of the nervous system. Habitual association is the continual association of two things until they become an indissoluble whole, such as months and days in columns of an almanac, etc. Privileged association plays an enormous rôle. Things become indissolubly fixed in our memory and thought, not because they are often in the field of reality, but because of a time, perhaps an only time, when the thing struck us and left an indelible trace in our nervous tissue. Such are the visions of our early childhood which have submerged the other memories of the same period, and no reason can be assigned why such remained in preference to a thousand other scenes. Krohn gives quite an extensive review of the theories concerning Pseudo-Chromesthesia, or colored-hearing. He claims that none of the many physical explanations nor the psychical association is comprehensive enough to explain all of the facts. However, we infer that he does not consider number-forms a closely allied

phenomenon, styling them *automatic* associations. He considers such explainable by psychic associations.[1]

Having presented the main body of facts, we may well ask, what does all this mean? Have we an exceptional phenomenon, unrelated to any other psychic activity, for which some special explanation must be given? Concerning the general explanations of *Synopsie* above presented, we have little to say, but it appears certain that these visual diagrams are only less ordinary examples from a much wider field of mental phenomena.

One of the most striking things about number-forms is their extremely early origin, yet that they have a period of gradual formation and development is also evident. Forms for the Greek alphabet, historic dates, the years that one has lived, months, and in a few cases, numbers, have been developed late in life. As a rule they are not so vivid and enduring, but there are striking exceptions. Mr. G. says, "My form resulted from an evening reverie while looking in the fire; and after studying higher mathematics, I added minus and plus infinity. I was about 14 when it originated." We must then conclude that forms may originate late in life, and that they certainly become more elaborate by use and time.

There is no more reason for isolating these mental activities from a much larger field than there is for isolating exceptional cases of memory or imagination from these general powers of the mind. In any new line of investigation, the exceptional always receives attention first. This seems to have been the case with Galton, and most of the others mentioned have followed largely in the same path. Beside the more complicated cases which we have been treating, we find more than as many persons in whose minds the numbers simply take a distinct direction. Sometimes the numbers are seen on a straight line running only to the right, left, or up, etc. Sometimes only the numbers appear. Again there is only the sensation of following in some particular direction whenever numbers are thought of; sometimes only a feeling that large numbers are far off. Prof. Story, in conversation with the writer, denied that he had a number-form, but finally remarked that large numbers appeared far off, although he saw nothing, and had no feeling of any particular direction; yet if 7 and 69 were thought of, there would appear to be room between for the other numbers. I felt sure that investigation on this point would throw light upon the subject, and at once began a re-examination of those who had denied

[1] AMERICAN JOUR. PSY., Vol. V, p. 20.

having a form, and others who thought they had none. Two hundred and fifty adults have been reached, and two hundred and ten have a feeling that numbers in some way recede from them. Many report that they have an upward movement. Mr. F. said he could not resist thinking of numbers as going up, the large ones getting very high. With others they appear to go straight in front, or at angle of 45 degrees. To many, large numbers simply appear to be far away, and they are unable to designate any direction. Mr. B., a man of mathematical ability, said he had no number-form, but his number series contracted. Beginning with zero it is more than twice as far to 100 as it is from 100 to 200, and so on towards infinity. This is exactly the law that can be seen in every number-form, though not in any fixed ratio. I believe *that nearly all persons possess some idea of extension of numbers, more or less indefinite.* At first I had no idea that any similar phenomenon could be found in my mental activity, but when I think of 99 in its relation to 1, the form appears about two yards in front of me at an angle of about 45°; and I have never been able by an effort to think of it otherwise except for a moment. I find that but few have even noticed how they cast numerical thought into space relations. Out of 480, above the 7th grade, 74% visualize all mental operations with numbers in some way; and but few had thought it could be otherwise. The suggestions in former studies led us to an examination of children. In the 1st, 2nd and 3rd grades 785 children were asked to count and then to tell if the numbers take any direction. The most unexpected directions were asked for first. Thirty-four said they went down; 60, to the right; 29, to the left; 74, right forward; 470, some upward movement. Of course none of these were considered with the number-forms, but we evidently have the same material here and many of these are probably as distinct as the complex forms.[1]

Here we see the dominance of the eye over the other senses. In most of the forms it appears that there is a tendency to keep within the field of vision; only in a few cases are the forms located outside of that field, and many seem to turn in order to avoid getting out of it.

The number series is a succession, the rudimentary concept of which dates back further than the actual numeration of objects. When children begin to count they usually represent this series by nods or movements of some kind. Many adults unconsciously make these movements. A girl, 17,

[1] It would be extremely interesting to know whether those blind from birth make use of any such space imagery.

always moves her great toe. She says it is irresistible. If these movements are suppressed we might still find them represented in thought in the form of a space series. Without doubt children tend to connect some movement or extension in space with numbers, and it is here that we are to find the genesis of number-forms. Back of any visual image seen by the mind's eye is the motor element in thought, which must have space as a back-ground. Some say they cannot think of any series of objects, as man, dog, cow, horse, etc., without some idea of succession in space. Infinity usually suggests a never-ending line, the direction of which is often definitely located. Many, in thinking of the distance between two places as so many miles, see it in space.

Can early association explain this general tendency to cast the number series into spatial form? Is there anything inherent in the number concept which in any way determines the association? What is the relation of geometry and numbers? Numbers are generally defined as a series of successions, and a number is said to have no relations except that it comes after, and is followed by another, while a geometrical figure may have several relations.

Euclid did not make that wide separation of geometry and arithmetic now so commonly practiced, but there is a general tendency to base primary mathematics again more and more on geometry. Sylvester said, "Every time I go deep enough I find a geometrical bottom."[1] Again we may ask: Are the facts herein set forth to be traced to the general fusion of sensations received through different senses? If so, then this subject is a part of a still larger one, including Pseudo-Chromesthesia, dramatization of numbers (that is, the giving to certain numbers the characteristics of certain persons, animals, or objects), and the whole range of facts which Wundt includes in *complicative association*. The more we attempt to trace any set of sensations or perceptions to their ultimate origin, the more this confusion or mingling is encountered. But it hardly seems best to class these phenomena under the same head. It is better to seek the explanation in the motor and space elements in thought. If I have succeeded in showing that number-forms and others are not such isolated phenomena as they have been considered, and that they have their genesis in simpler mental activity, the object of this short paper is accomplished. Local relationship is very dominant in all thought, and when we suppose we think abstractly, as a rule, we use, either consciously or unconsciously, some substitute. Such thoughts are at bottom symbolic, and in so far concrete.

[1] "Address before the British Association."

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